Report

Evaluate

$\boxed{2}$ $\mathcal{L}^{-1}\left(\dfrac{\sinh sx}{s^2 \cosh sa}\right)$

Sol

$$F(s)e^{st} = \frac{\sinh(sx)}{s^2\cosh(sa)}e^{st}$$

$s^2 = 0$, $\cosh(sa) = 0$

$$\cosh sa = \frac{e^{sa} + e^{-sa}}{2} = 0$$

$e^{sa} + e^{-sa} = 0 \implies e^{2sa} = -1$

$2sa = \ln(-1)$

$\ln(x+iy) = \ln(r) + i(\theta + 2n\pi)$

$x = -1$, $y = 0$, $r = 1$, $\theta = \pi$

$2sa = 0 + i(\pi \pm 2n\pi)$

$$\boxed{s_n = \frac{i(\pi \pm 2n\pi)}{2a}}$$

$$\operatorname*{Res}_{s=0} F(s)\,e^{st} = \lim_{s\to 0} (s-0)^2 \frac{\sinh(sx)}{s^2 \cosh(sa)} e^{st} = 0$$

$$\operatorname*{Res}_{s=s_n} F(s)\,e^{st} = \lim_{s\to s_n} (s-s_n) \frac{\sinh(sx)}{s^2 \cosh(sa)} e^{st}$$

$$= \lim_{s\to s_n} \frac{\sinh(sx) * e^{st}}{s^2} * \lim \frac{(s-s_n)}{\cosh(sa)}$$

$$= \frac{e^{s_n t} * \cancel{\cosh}\ \sinh(x s_n)}{s_n^2} * \lim_{s\to s_n} \frac{1}{\sinh(sa)\,\frac{1}{2sa}}$$

$$= \frac{e^{s_n t} * \sinh(x s_n)}{s_n^2} * \lim_{s\to s_n} \frac{2sa}{\sinh(sa)}$$

$$= \frac{e^{s_n t} \sinh(x s_n)}{s_n^2} * \frac{2 s_n a}{\sinh(s_n a)} \longrightarrow (1)$$

$$\sinh(x s_n) = i \sin\left[\frac{\pi \pm 2n\pi}{2a} x\right]$$

$$\sinh(a s_n) = i \sin\left[\frac{\pi \pm 2n\pi}{2\cancel{a}} \cancel{a}\right]$$

بالتعويض في (1)

$$\therefore f(t) = 0 + \frac{e^{s_n t} * i \sin\left(\frac{\pi \pm 2n\pi}{2a} x\right)}{s_n^2} *$$

$$\frac{2 s_n a}{i \sin\left(\frac{\pi \pm 2n\pi}{2}\right)}$$

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(3) $\mathcal{L}^{-1}\left(\frac{\cosh sx}{s^3 \cosh sb}\right)$

$$F(s) e^{st} = \frac{\cosh sx}{s^3 \cosh sb} e^{st}$$

$$s^3 = 0 \Rightarrow s = 0 \quad , \quad \cosh sb = 0$$

$$\cosh(sb) = \frac{e^{sb} + e^{-sb}}{2} = 0 \Rightarrow e^{sb} + e^{-sb} = 0 \quad * e^{sb}$$

$$e^{2sb} = -1 \Rightarrow 2sb = \ln(-1)$$

$$\ln(x+iy) = \ln(r) \pm i\,(\theta \pm 2n\pi)$$

$$x = -1 , \; y = 0 , \; r = 1 , \; \theta = \pi$$

$$2sb = 0 + i\,(\pi \pm 2n\pi)$$

$$s_n = \frac{i\,(\pi \pm 2n\pi)}{2b}$$

$$\operatorname*{Res}_{s=0} F(s)\, e^{st} = \lim_{s \to 0} (s-0)^3 \frac{\cosh sx \; e^{st}}{s^3 \cosh(sb)} = \boxed{1}$$

$$\operatorname*{Res}_{s=s_n} F(s)\, e^{st} = \lim_{s \to s_n} (s - s_n) \frac{\cosh sx}{s^3 \cosh(sb)} e^{st}$$

$$= \lim_{s \to s_n} \frac{e^{st} \cosh(sx)}{s^3} * \lim_{s \to s_n} \frac{(s - s_n)}{\cosh(sb)}$$

$$= \frac{e^{s_n t} \cosh(s_n x)}{s_n^3} * \lim \frac{1}{\sinh(sb)\frac{1}{2sb}}$$

$$= \frac{e^{s_n t} \cosh(s_n x)}{s_n^3} * \lim \frac{2sb}{\sinh(sb)}$$

$$= \frac{e^{s_n t} \cosh(s_n x)}{s_n^3} * \frac{2 s_n b}{\sinh(s_n b)} \longrightarrow \boxed{1} \quad (1)$$

$$\cosh(s_n x) = \cos\left[\frac{\pi \pm 2n\pi}{2b} x\right]$$

$$\sinh(s_n b) = i \sin\left(\frac{\pi \pm 2n\pi}{2}\right)$$

بالتعويض في (1)

$$f(t) = 1 + \frac{e^{s_n t} * \cos\left[\frac{\pi \pm 2n\pi}{2b} x\right]}{s_n^3} * \frac{2 s_n b}{i \sin\left(\frac{\pi \pm 2n\pi}{2}\right)}$$

$\boxed{3}$

$$\mathcal{L}^{-1} \frac{\tan \frac{as}{2}}{s(s+b)}$$

لم أستطع حلها .

$\boxed{6}$